Int. J. Heat Mass Transfer. Vol. 16, pp. 1319–1322. Pergamon Press 1973. Printed in Great Britain

SHORTER COMMUNICATIONS

TRANSIENT TEMPERATURE FIELD IN A PLANE SLAB WITH EMBEDDED CYLINDRICAL SOURCES

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(*Received* 2 *February* 1972 *and in revised form* 4 *October* 1972)

NOMENCLATURE

a_i, b_i, c_i,	steady state temperature coefficients;
A_k,	coefficient defined by equation (17);
B,	domain boundaries;
Bi,	Biot number, hR_0/k;
D,	solution domain;
$f(\boldsymbol{R})$,	initial condition;
$F(\boldsymbol{R})$,	function given by equation (10);
h,	film coefficient;
H_1, H_2,	slab dimensions shown in Fig. 1;
k,	thermal conductivity of slab material;
L,	heater half spacing;
p_i,	coefficients given by equation (3);
q,	heat flux;
Q,	dimensionless heat flux, qR_0/kt_0;
r,	dimensionless radial coordinate;
R_0,	heater radius;
$\boldsymbol{R}$,	position vector;
t,	temperature;
T,	dimensionless temperature, t/t_0;
U,	transient solution for homogeneous boundary conditions;
V,	steady state solution
x, y, z,	cartesian coordinates;
α,	thermal diffusivity;
ϕ,	angular coordinate;
λ,	eigenvalue;
θ,	dimensionless time (Fourier modulus), $\alpha\tau/R_0^2$;
τ,	time;
η,	dimensionless outward directed normal.

Indices

0,	heater surface;
1,	slab upper surface;
2,	slab lower surface.

INTRODUCTION

THE PROBLEM of pavement heating installations can be simulated by a plane slab with embedded heat sources [1]. In cases where the heater diameter is small compared to the heater depth, an analytical solution can be obtained assuming the heaters to be point sources in a two dimensional domain. However, when dealing with installations consisting of buried tubes with a heated fluid flowing inside, the heater diameter is about the same size as the heater depth, as suggested by the *ASHRAE Guide and Data Book* [2]. In this situation, a better model for the problem is a plane slab with cylindrical holes which are constant temperature surfaces [1], there is no analytical solution to this problem and an approximate solution was obtained using the point-matching technique.

The point-matching technique has been successfully used to solve steady state heat conduction problems in solids bounded internally by a cylinder [3–5]. Ojalvo and Linzer [6] presented a good description of the method and different techniques to improve the solution; a brief explanation of the use of point-matching in transient problems was given, but only a one dimensional example was solved. Sparrow *et al.* [7] developed a new method to generate a family of functions which are solutions of the transient heat transfer equation, using a cartesian coordinate system. In [8] it was found that a polar coordinate system, as suggested in [6], and the least square fit technique used by Sparrow [7], would produce more accurate results with less computing time, when applied to the problem in question.

* The material for this paper was developed in partial fulfillment of the requirements for the degree, Doctor of Philosophy, Mechanical Engineering, Department, Kansas State University, Manhattan, Kansas.

FORMULATION

The heat flow in a homogeneous solid with temperature independent properties is governed by:

$$\nabla^2 t = \frac{1}{\alpha}\frac{\partial t}{\partial \tau}. \qquad (1)$$

Using the dimensionless form and assuming linear boundary conditions, results in:

$$\nabla^2 T = \frac{\partial T}{\partial \theta}, \qquad \text{in } D \qquad (2)$$

$$p_1 \frac{\partial T}{\partial \eta} + p_2 T = p_3, \qquad \text{on } B \qquad (3)$$

$$T = f(\boldsymbol{R}), \qquad \text{in } D, \theta = 0. \qquad (4)$$

Where $p_i (i = 1, 2, 3)$ are constants or functions of the space variables only, and η is the outward directed normal to the boundaries.

In order to apply the point-matching technique the solution is divided into two parts.

$$T(\boldsymbol{R}, \theta) = U(\boldsymbol{R}, \theta) + V(\boldsymbol{R}) \qquad (5)$$

where U is the transient solution for homogeneous boundary conditions and V is the steady state solution.

Therefore, equations (2)–(4) are transformed into

$$\nabla^2 V = 0 \qquad \text{in } D \qquad (6)$$

$$p_1 \frac{\partial V}{\partial \eta} + p_2 V = p_3 \qquad \text{on } B \qquad (7)$$

$$\nabla^2 U = \frac{\partial U}{\partial \theta} \qquad \text{in } D \qquad (8)$$

$$p_1 \frac{\partial U}{\partial \eta} + p_2 U = 0 \qquad \text{on } B \qquad (9)$$

$$U = F(\boldsymbol{R}) = f(\boldsymbol{R}) - V(\boldsymbol{R}) \qquad \text{in } D, \theta = 0. \qquad (10)$$

To solve the problem in a realistic manner, the following assumptions should be added [1]:

1. Temperature variation in the direction of the heater axis is negligible.
2. Heater spacing and depth are uniform.
3. Thermal resistance between heater and slab is negligible.
4. Boundary conditions are independent of time.
5. The slab initially has an arbitrary temperature distribution and for $\theta = 0$ the dimensionless temperature around the heater circumference jumps to one and is kept at this value throughout the process.

With these assumptions, the solution domain D and the coordinate system are those shown in Fig. 1. The values of the p_i in equations (3), (7) and (9) are given below:

$$\left.\begin{array}{l} p_1 = 1 \\ p_2 = Bi_2 \\ p_3 = -Q_2 \end{array}\right\}, \qquad x = H_2, \quad 0 \leqslant y \leqslant L \qquad (11)$$

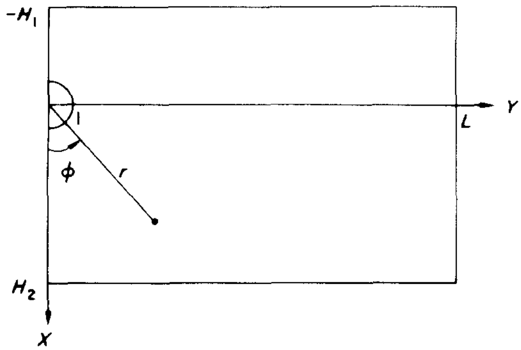

FIG. 1. Coordinate system for homogeneous slab with embedded cylindrical heat sources.

$$\left.\begin{array}{l} p_1 = 1 \\ p_2 = 0 \\ p_3 = 0 \end{array}\right\}, \qquad y = L, \quad -H_1 \leqslant x \leqslant H_2 \qquad (12)$$

$$\left.\begin{array}{l} p_1 = 1 \\ p_2 = Bi_1 \\ p_3 = -Q_1 \end{array}\right\}, \qquad x = -H_1, \quad 0 \leqslant y \leqslant L \qquad (13)$$

$$\left.\begin{array}{l} p_1 = 0 \\ p_2 = 1 \\ p_3 = 1 \end{array}\right\}, \qquad r = 1, \quad 0 \leqslant \phi \leqslant \pi. \qquad (14)$$

STEADY STATE SOLUTION

The general solution of equation (6) obtained by separation of variables (3) and using symmetry is

$$V(r, \phi) = a_0 + a_3 \ln r + \sum_{n=1}^{\infty} (b_n r^n + c_n r^{-n}) \cos n\phi. \qquad (15)$$

From the conditions given by equation (14), and truncating the series in equation (15) for $n = N$, results in

$$V(r, \phi) = 1 + a_3 \ln r + \sum_{n=1}^{N} b_n (r^n - r^{-n}) \cos n\phi. \qquad (16)$$

The coefficients a_3, b_1, $b_2, \ldots b_N$ are determined by point-matching, as explained in [8].

TRANSIENT SOLUTION

The transient solution is given in [7, 8] as

$$U(r, \phi, \theta) = \sum_{k=1}^{\infty} A_k \exp(-\lambda_k^2 \theta)\, W_k(r, \phi) \qquad (17)$$

where, as in [8]

$$W_k(r, \phi) = \sum_{n=0}^{\infty} D_{nk} G_{nk}(r, \phi, \lambda_k) \qquad (18)$$

$$G_{nk}(r, \phi, \lambda_k) = \left[J_n(\lambda_k r) - \frac{J_n(\lambda_k)}{Y_n(\lambda_k)} Y_n(\lambda_k r) \right] \cos n\phi \qquad (19)$$

where J_n and Y_n denote the Bessel functions of order n and of the first and second kind, respectively.

The series are truncated for application of the point-matching technique as explained in [6–8]. The boundary conditions are used to determine the eigenvalues λ_k and coefficients D_{nk}. The coefficients A_k are calculated from the initial conditions.

The complete solution is then:

$$T(r,\phi,\theta) = \sum_{k=1}^{K} A_k \exp(-\lambda_k^2\theta)\, W_k(r,\phi) + 1 + a_3 \ln r + \sum_{n=1}^{N} b_n(r^n - r^{-n}) \cos n\phi. \qquad (20)$$

NUMERICAL EXAMPLE:

Consider the slab and heater arrangement such that (with heater radius = 1)

$$H_1 = 5$$
$$H_2 = 10$$
$$L = 12.$$

Initially the whole slab is at the uniform temperature $T = 0$. At the instant $\theta = 0$ the heaters are turned on and the dimensionless heater surface temperature jumps to 1 and is kept at this value. The lower surface is insulated and the upper surface loses heat by convection to the environment at 0 degrees, with a Biot number $Bi_1 = 0{\cdot}127$ and dimensionless heat flux $Q_1 = 0$.

The first three eigenvalues are listed below, as well as the eigenvalues for the point source approximation for the same geometry and boundary conditions [9].

Eigenvalues	Example	Point source
λ_1	0·1301	0·07081
λ_2	0·2523	0·2417
λ_3	0·304	0·2712

Figure 2 shows the variation of the temperature profile on the upper surface, with time.

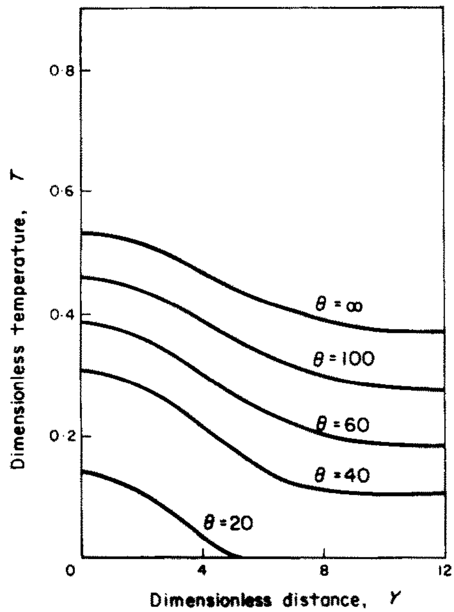

FIG. 2. Upper surface dimensionless temperature profile variation, with time, during heating.

CONCLUSIONS

The point-matching technique seems to be an excellent means of solving the class of problems discussed in this work when used with a digital computer. The part that consumes the most computer time is the calculation of the eigenvalues because of the complicated transcendental equations that must be solved by trial and error. However, since both the eigenvalues and the coefficients D_{kn}'s depend solely on the homogeneous boundary conditions, different initial conditions can be analyzed quickly at very little extra cost since only the coefficients A_k's need to be recalculated. The same reasoning applies in cases where only the nonhomogeneous part of the boundary conditions change, namely Q_1 and Q_2. In this case only the steady state solution $V(r,\phi)$ and the coefficients A_k's have to be re-evaluated.

It can be seen from the results that the point source approximation cannot be used for situations similar to those in the numerical example, as explained in the introduction. The eigenvalues of the point source approximation are lower bounds for the real case.

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Int. J. Heat Mass Transfer. Vol. 16, pp. 1322–1324. Pergamon Press 1973. Printed in Great Britain

EFFECT OF ELECTRIC FIELD ON BOILING HYSTERESIS IN CARBON TETRACHLORIDE

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(*Received* 24 *April* 1972 *and in revised form* 1 *December* 1972)

NOMENCLATURE

E_w, electric field intensity at the wire surface [V/m];
g_e, effective gravitational acceleration [m/s^2];
κ, relative permittivity of the liquid [dimensionless];
r_c, internal radius of the outer cylinder [m];
r_w, outer radius of the heater wire [m];
v, voltage applied between the wire and the outer cylinder [V];
ϵ_0, permittivity of free space [F/m];
ρ_L, density of the liquid [kg/m^3].

INTRODUCTION

THE INCREASE of heat-transfer rate in a fluid, due to the application of an electric field was first pointed out by Senftleben [1]. Ahsmann and Krönig [2] studied the effect of electric field of rectangular waveform on the heat-transfer of some organic liquids. Several workers tried to explain the electro-convectional heat-transfer as due to the change of the electric susceptibility in presence of a temperature gradient. According to Weber and Halsey [3] the heat-transfer rate is suppressed by one of the non-conservative forces in the fluid acting towards the heat source due to the interaction of thermal and electrical gradients. They also concluded that this effect is small compared with the improved heat-transfer due to the motion of the free charges under the action of an electric field.

Allen [4] applied both d.c. and a.c. fields separately and also studied the effect of mixed stresses on the electro-convectional heat-transfer. According to him the alternating stress alone can increase the heat-transfer rate and enhancement of the heat-transfer due to the application of the unidirectional stress, as reported by early workers, was not due to the unidirectional field but due to the "ripples" present in the high voltage supplies. Like Weber and Halsey [3] he also suggested that the increase in heat-transfer is due to the motion of electric charges, present in the liquid bulk, under the action of alternating electric field. Watson [5] experimented with n-hexane. He disregarded the idea that the effect of electric field on the free charges is the reason for enhancement of the heat-transfer. According to him a permittivity gradient is created due to the temperature gradient in the liquid and the increased field strength enhances heat-transfer rate from the heater, since the force on the non-homogeneous dielectric increases with the square of the electric field.

Mascarenhas [6] was of the same opinion that the change in the heat-transfer rate in a liquid dielectric is due to the interaction between the thermal and the electric fields. The thermal conductivity of liquid shows strong variation due to the action of the electric field.

According to Markels and Durfee [7] the increase in the heat-transfer is due to "Dielectrophoresis"—a phenomenon in which there is a movement of the dielectric liquid when placed in a non-uniform electric field and this movement is caused by induced polarization.

Choi [8] investigated the effect of the radial electric field on boiling heat-transfer in a dielectric liquid. He applied d.c. field and found improvement in heat transfer with increasing fields. Choi concluded that, in a non-uniform